Master method –

$$T(n) = aT\left(\frac{n}{b}\right) + f(n)$$

$a \geq 1 \quad b > 1 \quad f(n) = \theta(n^k \log^p n)$

– Find b^k

– case 1 – if $a > b^k \rightarrow T(n) = \theta(n^{\log_b^a})$

case 2 – if $a = b^k$

a) $p > -1$ Then $T(n) = \theta(n^{\log_b^a} \log^{p+1} n)$

b) $p = -1$ $T(n) = \theta(n^{\log_b^a} \log \log n)$

c) $p < -1$ $T(n) = \theta(n^{\log_b^a})$

case 3 – if $a < b^k$

a) if $p \geq 0$ $T(n) = \theta(n^k \log^p n)$

b) if $p < 0$ $T(n) = O(n^k)$

1- $T(n) = 3T(\frac{n}{2}) + n^2$

$a = 3 \quad b = 2 \quad f(n) = \theta(n^k \log^p n)$

$k = 2 \quad p = 0$

$b^k = 2^2 = 4$

$a \equiv b^k$

$3 < 4$

$a < b^k$

Case 3a - $a < b^k$ $\quad p = 0$

$T(n) = \theta(n^k \log^p n)$

$= \theta(n^2 \log^0 n)$

$\boxed{\therefore T_n = \theta(n^2)}$

2- $T(n) = 4T\frac{n}{2} + n^2$

$a = 4 \quad b = 2 \quad k = 2 \quad p = 0$

$b^k = 2^2 = 4$ and a is 4 so

Case 2 - $a = b^k$

Now $p = 0$ i.e $p > -1$

so $\cancel{T(n) = \theta(n^{\log} \ldots}$

$$T(n) = \theta(n^{\log_b a} \log^{p+1} n)$$

$$T(n) = \theta(n^{\log_2 4} \log^{0+1} n)$$

$$= T(n) = n^{2 \cdot \log_2 2} \log n)$$

$$= \boxed{T(n) = n^2 \log n}$$

$\because \log_2 2 = 1$

3. $T(n) = T(\frac{n}{2}) + n^2$

$a = 1$ $b = 2$ $k = 2$ $p = 0$

$b^k = 2^2 = 4$

$a < b^k$

Case <u>3 a</u> $p \geq 0$ $T(n) = \theta(n^k \log^p n)$

$$T(n) = \theta(n^2 \cdot \log^0 n) = \theta(n^2)$$

4- $T(n) = 2^n T\left(\frac{n}{2}\right) + n^n$

$a = 2^n \quad b = 2 \quad k = n \quad p = 0$

Hear we can not ~~aff~~ apply master's method to solve this

5- $16\, T\left(\frac{n}{2}\right) + n$

$a = 16 \quad b = 2 \quad k = 1 \quad p = 0$

$b^k = 2^1 = 2$

$a > b^k \quad p \geq -1$

Case 1 – $T(n) = \theta\left(n^{\log_b^a}\right)$

$T(n) = \theta\left(n^{\log_2^{16}}\right)$

$= \theta\left(n^{\log_2 2^4}\right) = \theta\left(n^4\right)$

6- $T(n) = 2T\frac{n}{2} + n\log n$

$a = 2 \quad b = 2 \quad k^1 \quad p = 1$

$b^k = 2^1 = 2$

$a = b^k \quad p = 1$ i.e $p > -1$

case 2 a – $T(n) = \theta\left(n^{\log_b^a} \log^{p+1} n\right)$

$$T(n) = \theta(n^{\log_2 2} \log^{1+1} n)$$

$$\boxed{T(n) = \theta(n \log^2 n)}$$

7- $T(n) = 2T(\frac{n}{2}) + \frac{n}{\log n}$

$$= 2T\frac{n}{2} + n^1 \log^{-1} n)$$

$a = 2 \quad b = 2 \quad k = 1 \quad p = -1$

$$b^k = 2^1 = 2$$

$a = b^k \quad p = -1$

cas 2b - $T(n) = \theta(n^{\log_b a} \log \log n)$

$$T(n) = \theta(n^{\log_2 2} \log \log n)$$

$$\underline{T(n) = \theta(n \log \log n)}$$

8- $T(n) = 2T\frac{n}{4} + n^{0.51}$

$a = 2 \quad b = 4 \quad k = 0.51 \quad p = 0$

$b^k = 4^{0.51} \qquad a < b^k \quad p \geq 0$

case 3a - $T(n) = \theta(n^{.51} \log^0 n) = \theta(n^{.51})$