Master method ---->

1. $6T\frac{n}{3} + n^2 \log n$

$a = 6 \quad b = 3 \quad k = 2 \quad p = 1$

$b^k = 3^2 = 9$

$a < b^k$ and $p = 1$

Case 3 a - $T(n) = \theta(n^k \log^p n)$

$T(n) = \theta(n^2 \log^1 n)$

$T(n) = \theta(n^2 \log n)$

2- $T(n) = 7T\frac{n}{3} + n^2$

$a = 7 \quad b = 3 \quad k = 2 \quad p = 0$

$b^k = 3^2 = 9$

$a < b^k$ and $p \geq 0$

Case 3 a - $T(n) = \theta(n^k \log^p n)$

$= \theta(n^2 \log^0 n)$

$= \theta(n^2)$

3- $T(n) = 4T\frac{n}{2} + \log n$

$a=4 \quad b=2 \quad k=0 \quad p=1$

$b^k = 2^0 = 1$

$a > b^k$ and ~~$b > -1$~~

Case 2a $- T(n) = \Theta(n^{\log_b^a} \log^{p+1} n)$

$T(n) = \Theta(n^{\log_2^4} \cdot \log^{1+1} n)$

$= \Theta(n^2 \cdot \log^2 n)$

Case 1 a $-$

$T(n) = \Theta(n^{\log_b^a})$

$= \Theta(n^2)$

4- $T(n) = \sqrt{2}\, T\frac{n}{2} + \log n$

$= 2^{\frac{1}{2}} T\frac{n}{2} + \log n$

$a = \sqrt{2} \quad b = 2 \quad k = 0 \quad p = 1$

$b^k = 1$

$a > b^k$

Case 1 $-$

$T(n) = \Theta(n^{\log_b^a})$

$= T_n = \Theta(n^{\frac{1}{2}})$

$= \Theta(\sqrt{n})$

$\log_2^{\sqrt{2}}$

$\frac{1}{2} \log_2^2 = \frac{1}{2}$

5- $T(n) = 3T\frac{n}{2} + n$

$a=3 \quad b=2 \quad k=1 \quad p=0$

$b^k = 2^1 = 2$

$a > b^k$

Case 1 $-$

$T(n) = \Theta(n^{\log_b^a})$

$= \Theta(n^{\log_2^3})$

6- $T(n) = 3T\frac{n}{3} + \sqrt{n}$

$a = 3 \quad b = 3 \quad k = \frac{1}{2} \quad p = 0$

$b^k = 3^{\frac{1}{2}}$

$a > b^k$ — case 1 — $T(n) = \theta(n^{\log_b^a})$

$T(n) = \theta(n^{\log_3^3}) = \theta(n)$

7- $T(n) = 3T\frac{n}{4} + n\log n$

$a = 3 \quad b = 4 \quad k = 1 \quad p = 1$

$b^k = 4^1 = 4$

$a < b^k \quad p \geq 0$

case 3a

$T(n) = \theta(n^k \log^p n)$

$T_n = \theta(n \log n)$

8- $T(n) = T\sqrt{n} + 1$

$n = 2^m \qquad m = \log_2 n$

$T(2^m) = T(2^m)^{1/2} + 1$

$= T(2^{m/2}) + 1$

$S(m) = T\left(\frac{m}{2}\right) + 1$

$a = 1 \quad b = 2 \quad k = 1 \quad p = 0$

$b^k = 2^1 = 2$

$a < b^k \quad p \geq 0$

Case 3 a-

$T(n) = \theta(n^k \log^p n)$

$T(n) = \theta(n \log^0 n) = \theta(n)$

$T(n) = \theta(\log \frac{n}{2})$

comparision of time complexities —

$O(c) < O(\log\log n) < O(\log n) < O(n^{1/2}) < O(n)$

$O(n) < O(n \log n) < O(n^2) < O(n^3) < O(n^k)$

$O(n^k) < O(2^n) < O(n^n) < O(2^{2^n})$

Ex — compare n and $(\log n)^{100}$