

Magic Squares

There is a general pattern for making magic squares with an odd number of rows.
The rules are:

1. Arrange the numbers from smallest to largest.
2. Place the smallest number in the middle square on the top row.
3. Put succeeding numbers on a diagonal (going to the right and up).
4. When you run out of rows (fall off the top), go to the bottom of the column in which you want to place the number.
5. When you run out of columns (fall off to the right), go to the extreme left of the row in which you want to place the number.
6. If the space in which you want to place the number is occupied, put the number in the space below the last occupied space.
7. When you fall out the main diagonal, put the number in the space below the last occupied space (which should be in the second row on the extreme right).
8. If you are using consecutive numbers beginning with 1, you can find the magic number by the formula, $(n(n^2 + 1) / 2)$, where n is the number of rows.

Examine the Magic Square below. See how it was created using the rules above.

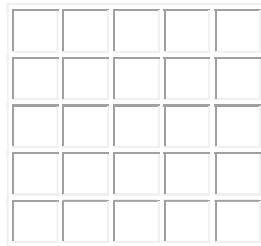
8	1	6
3	5	7
4	9	2

Here is how the magic square was constructed:

1. Arrange the numbers from smallest to largest: 1, 2, 3, 4, 5, 6, 7, 8, 9.
2. Place the smallest number in the middle square on the top row:
1 goes in the middle square on the top row.
3. Put succeeding numbers on a diagonal (going to the right and up):
5, and 6 go in a diagonal to the right and up.
4. When you run out of rows (fall off the top), go to the bottom of the column in which you want to place the number:
2 runs out the top, so place it at the bottom of the 3rd column.
9 also runs out the top and is placed at the bottom of the 2nd column.
5. When you run out of columns (fall off to the right), go to the extreme left of the row in which you want to place the number:
3 falls off the right, so it is placed in the extreme left square of row 2.
8 falls off the right, so it is placed in the extreme left square of row 1.

6. If the space in which you want to place the number is occupied, put the number in the space below the last occupied space:
4 should go where 1 is, but since it is occupied, place 4 below the number 3.
 7. When you fall out the main diagonal, put the number in the space below the last occupied space (which should be in the second row on the extreme right):
7 falls out the main diagonal, so it is placed below the 6 (in the extreme right cell of the second row).
 8. If you are using consecutive numbers beginning with 1, you can find the magic number by the formula, $(n(n^2 + 1) / 2)$, where n is the number of rows.
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Now try your hand at filling in the squares in a 5x5 square using the numbers 1, 2, 3, 4, ... 25. Make it a magic square. What is the magic number?



Magic square of 3

		$i \rightarrow$			
$j \downarrow$	0	2	7	6	$\rightarrow 15$ $= 15$
	1	9	5	1	
	2	4	3	8	
					$\downarrow$ 15

No. of element $\Rightarrow n^2$

$$\text{i.e.} \Rightarrow 3^2 = 9$$

i.e

1, 2, 3, 4, 5, 6, 7, 8, 9

Fill these value in the matrix
in such a way that sum of diagonal
element, sum of any rows or column
should be $\frac{n(n^2+1)}{2}$

$$\text{i.e.} \quad \frac{3(3^2+1)}{2} = 3 \times \frac{10}{2} = 15$$

Steps of filling the cell $\rightarrow$

- ① Arrange the numbers in increasing order

1 - . . . - - - n^2

1 2 3 4 5 6 7 8 9

- ② Find the position of number

- ③ initially $(i, j) \Rightarrow (\frac{n}{2}, n-1)$
next position $\Rightarrow (i-1, j+1)$

- ④ if $i = -1 \rightarrow i = n-1$

- ⑤ if $j = n \rightarrow j = 0$

- ⑥ if occupied $i = i+1, j = j-2$

- ⑦ if i and j both are occupied
if $(-1, n) \Rightarrow (0, n-2)$

	$i \rightarrow$	0	1	2
$j \downarrow$	0	2	7	6
	1	9	5	1
	2	4	3	8

$n=3$

Current position	next position	Value	Remark
$(1, 2)$	—	1	$(\frac{n}{2}, n-1)$
	$(0, 3)$		$(i-1, j+1)$
$(0, 3)$	$(0, 0)$		Not define if $i=n, j=0$
$(0, 0)$	$(-1, 1)$	2	$i-1, j+1$
$(-1, 1)$	$(2, 1)$		if $i=-1, i=n-1$
$(2, 1)$	$(1, 2)$	3	$i-1, j+1$
$(1, 2)$	$(2, 0)$		occupied $i=i+1, j=j-2$
$(2, 0)$	$(1, 1)$	4	$i-1, j+1$
$(1, 1)$	$(0, 2)$	5	$i-1, j+1$
$(0, 2)$	$(-1, 3)$	6	$i-1, j+1$

$(-1, 3)$	$(0, 1)$		if $i = -1$ $j = n$ $i = 0$ $n - 2$
$(0, 1)$	$(-1, 2)$	7	$i - 1, j + 1$
$(-1, 2)$	$(2, 2)$		if $i = -1, i = n - 1$
$(2, 2)$	$(1, 3)$	8	$i - 1, j + 1$
$(1, 3)$	$(1, 0)$		if $j = n, j = 0$
$(1, 0)$	—	9	—